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Book Review

The Princeton Companion to Applied Mathematics by Nicholas J. Higham (ed.), Princeton University Press, Princeton and Oxford, 2015; ISBN: 978-0-691-15039-0

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What is Applied Mathematics? "Applied Mathematics consists in solving exact problems approximately and approximate problems exactly." (Kurt Friedrichs. In: Chelluri C. A. Sastri, *Pure and Applied Mathematics.* Notices of the AMS, April 2007, p. 471). In particular, the initial-boundary value problems associated with the Navier–Stokes equations and turbulence are the examples of issues that are notoriously difficult to solve exactly and where we look for the model, useful solutions. The practical definition may be as follows: Applied mathematics is a branch of mathematics that concerns itself with the mathematical techniques typically used in the application of mathematical knowledge in many spheres of science, industry, and everyday life.

This book follows the highly successful *Princeton Companion to Mathematics* [Gowers T. et al. (eds.), Princeton University Press, Princeton and Oxford, 2008], which concentrates on modern pure mathematics. If desired, the reader could consult this book, whose utility is certified by the many citations in the scientific literature.

The book is divided into eight parts encompassing 186 articles written by 165 authors.

Part I, Introduction to Applied Mathematics, a six articles in total, starts with an interesting tour of the relationship between pure and applied mathematics. Next, the basic notation, terminology and concepts of applied mathematics are briefly presented. The following chapter contains a terse presentation of methods and techniques used to solve applied problems. The concept of algorithms is explained shortly in the next chapter. Finally, the goals of applied mathematical research are described, together with a brief excursion of the history of applied mathematics.

Part II, Concepts, comprises 36 short articles. It provides the essential notions and tools required for many practical application of mathematics. The part begins with asymptotic expansions and ends with waves (arranged in alphabetical order). These include, among others: complex systems, chaos and singularities; conservation laws, invariants, tensors and manifolds; function spaces, integral transforms and convolution including a discussion of the fast Fourier transform; interval analysis and selected numerical methods; some matrix operations; modelling and model reduction as well as the probabilistic approach to uncertainty. The choice of topics and the order in which they are presented is very thoughtful.

Part III, Equations, Laws, and Functions of Applied Mathematics, contains thirty-one articles: twenty-four "Equations" and seven "Functions and Laws". Two of the articles I particularly liked are "The Lambert W Function" and "Einstein's Field Equations", which are very instructive and readable. Among the problems studied here are the second-order partial differential equations in the context of the classical mathematical physics (transport processes, various types of waves, heat flows, steady states in a homogeneous bodies, etc.). The relations of the differential equations to variational principles, mechanics of continua and boundary-value problems are briefly discussed and analysed. Two articles are on Schrödinger's equation and its relativistic counterpart—the Dirac equation. In addition two of the

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paradigmatic examples of systems with chaotic behaviour, namely, the logistic map and the Lorenz equations, are explored. The financial-statistical problems with particular reference to black–scholes equation are also included.

The core of the book is Part IV, which consists expositions of forty "Areas of Applied Mathematics" (417 pages). Eleven articles are about the equations and the various methods used to solve them, describing well-established methods and recent advances in the field of algorithm development for numerical modelling. The article on the algebraic geometry, is particularly interesting; it contains a variety of results that are not only restricted to pure mathematics, but also concentrates on the symbolic exact methods. There are five articles that could be classified as mathematical description of continuous media. They are the collection of research written by the leading experts who have made important contributions to these areas. Topics include: mechanics of continua, fluid mechanics, magnetohydrodynamics, mechanics of solids and general relativity. Another article presents the mathematical concepts relating to network analysis together with same applications to real-world problems. Geophysical aspects of applied mathematics are the subject of the three articles, and include seismic explorations and the various dynamical processes in the atmosphere and the oceans. The remaining articles address a wide variety of another important topics and techniques of applied mathematics: complex analysis, calculus of variations, spectral theory, information theory, combinatorics and graph theory, with particular attention to physical, and sometimes other applications of the main subjects (finance, biology and medicine, for example).

Mathematical modelling is the art of describing essential aspects of reality with the help of language of mathematics. Part V, Modelling, a 21 articles in total, is concerned with the various applications of mathematics, including authoritative articles on biology, medicine, epidemiology, finance, mechanical engineering, statistics, physics, fluid mechanics and geophysics. In particular, I read with great interest the article about *Inerters*, i.e. the ideal mechanical two-terminal elements, which are the substitutes for the mass element, where the applied force is proportional to the relative acceleration across the terminals. Five articles deal with non-linear evolution problems. Discussed are granular flows, sea ice dynamics, the weather prediction, tsunami modelling and the turbulence problem. The development of mathematically correct and physically admissible theories to describe and predict these complex behaviours is a topic of fundamental importance to both the earth sciences and applied mathematics. Two articles concern the developments in biomechanics. The hydrodynamics of a swimmer is described by an incompressible, turbulent, two-phase flow model and the blood arterial flows characterized by moderately high Reynolds numbers, are briefly discussed. The ways in which mathematics may be used to give insight into physiological, epidemiological and cardiological questions, and how there issues can lead to new mathematical problems, are presented also. The different aspects of financial mathematics and the importance of rational portfolio choice criteria complement this excellent part.

Part VI, Example Problems, is structured into eighteen short articles. The topics covered ranged from examples occurring in rheology, biology, mechanical engineering, etc., to applications in physics (for example, *N*-body problem). Recent theoretical and experimental progress on making objects invisible to detection by electromagnetic waves is described first. Thereafter come the articles dealing with the bubbles and foams (foam is a disperse system, consisting of gas bubbles, separated by liquid layers). The next topic adresses the classic control problem—an inverted pendulum. Two articles deal with objects flight. A terse history of research in insect flight and current findings in unsteady aerodynamics of flapping flight are described. In addition, a recent analysis for the aerodynamics of a smooth sphere in context of the flight of a golf ball is presented. The subsequent works concern the automatic differentiation and the mathematics of knotting and linking in molecular biology. Next article provides the mathematical concepts connected with web search and link analysis, while, succeeding, surveys the various applications of graph algorithms in everyday life. Two interesting articles concentrate on the algorithms allowing the computation of the elementary functions

and the methods for generation of the pseudorandom numbers. The remaining articles of the part are devoted to the more practical problems concerning with the study optimal sensor location in the control of energy-efficient buildings, the investigation of non-smooth processes such as impact, switching, sliding and rattling, robotics, and the travelling salesman problem. The final two chapters contain the fascinating results related to *N*-body problem in context of impact on astronomy and dark matter.

Part VII of the book is made up of articles about twenty-five "Application Areas". Both classical and lesser known fields are considered. The first article focuses on the basic aircraft noise problems. Another article presents shortly a Python-based software package called Geoduck, which is designed for hybrid symbolic-numeric computations in complex geometry-processing problems posed by Boeing. The next article treats the computer-assisted proofs in analysis using methods from interval arithmetic. The following article is devoted to the presentation of some methods for the solution of selected problems, which are based on the max-plus formulation for these problems. The max-plus algebra is a commutative semifield in which the addition operation is maximization, and the multiplication operation is defined as standard addition. The next, very interesting article, deals with the mathematical aspects of online social networks communication. The further article outlines some important issues relating to system-on-a-chip design. Three articles discuss image processing problems including the specifications and usage of colour spaces, as well as the basic mathematical and engineering issues connected with image processing in general, and medical imaging in particular. In addition, an important method to capture and represent compressible signals, called compressive sensing, is presented and elucidated. The subjects of three succeeding articles are crucial for science: the computer programming languages, high performance scientific computing, and scientific visualization. Two articles pertain to the applied physics problems, namely, the electronic structure of materials in context of solid state physics, and the flame propagation in a combustible mixture. Theoretical and practical aspects of seismic imaging, radar imaging and *X* -ray imaging detection techniques, are the themes of another articles. The nature of mathematical economics is exposed briefly, together with the foundations of mathematical neuroscience. Other articles discuss, among others, the fundamentals of systems biology, the basic concepts of communication networks, the field of text mining, and voting systems.

Part VIII, Final Perspectives, is about the mathematics in a broader context. It consists of nine articles which describe some elements of scientific workshop. These include the mathematical writing, critically and creatively reading and popularisation. Topics like: methodology, style as well as LaTeX suggestions are also addressed. My favourite part of the text is about the experimental applied mathematics, a remarkably terse survey of the rudiments of interesting things from an experimental-mathematical perspective, in just a few pages (for example, the Ising integrals). At last, the following questions are considered: how to teach applied mathematics; how are representations of mathematics and mathematicians in popular culture, and the mutual relationships between mathematics and politics.

Overall this is a very reliably and surprisingly uniformly written book. Almost all material in the book is accessible to graduate or even undergraduate students of exact and natural disciplines. Many articles are supplied with additional information which make the book practically self-contained. The book can be decisively recommended for beginners as well as for expert researchers working in the domain of applied mathematics.